SOLUTIONS

Christ Church
Grammar School

2018
UNIT TEST 6

MATHEMATICS METHODS Year 11

Section Two:
Calculator-assumed

Student name ______________________________

Teacher name ______________________________

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 34 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that **you do not use pencil**, except in diagrams.

Question 4 **(4 marks)**

The values of the first four terms of a geometric sequence are as follows:

$$6,\ 6\sqrt{2},\ 12,\ 12\sqrt{2}$$

Calculate the value of the first term in this sequence that exceeds 1400.

Sequence is a G.P.

with $a = 6$ ✓ (for first term)

$r = \sqrt{2}$ ✓ (for common ratio)

So

✓ (any one of these representations is OK) $T_n = 6 \times (\sqrt{2})^{n-1}$ or $T_{n+1} = \sqrt{2}\, T_n$, $T_1 = 6$ or $T_n = \sqrt{2}\, T_{n-1}$, $T_1 = 6$

From 'sequence' on Classpad: the value of the first term to exceed 1400 is

1536 ✓ (for final answer)

Question 5 **(7 marks)**

A school hall has 50 seats in row A, 54 seats in row B, 58 seats in row C and so on. That is, there are four more seats in each subsequent row.

(a) How many seats are in row K? (2 marks)

$T_n = a + (n-1)d$

$= 50 + 10 \times 4$ ✓(for using $n=11$)

$= 50 + 40$

$= 90$ seats ✓(for final answer)

(b) How many seats are there altogether if the back row is row Z? (2 marks)

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{26} = \frac{26}{2}(2 \times 50 + (26-1) \times 4)$ ✓(for correctly substituting into formula)

$= 13 \times 200$

$= 2600$ seats ✓(for final answers)

(c) The hall is extended by adding more rows at the back of the hall, taking the same pattern. If the final seating capacity of the hall is 3410, how many more rows were added? (3 marks)

$S_n = 3410$

$3410 = \frac{n}{2}(100 + (n-1) \times 4)$ ✓(for initial equation)

$6820 = 100n + 4n^2 - 4$

Solve on CAS $n = -55$ or 31 ✓(Solves for n and discounts $n = -55$)

discount $n = -55$ as $n > 0$ so $n = 31$ is solution

$\therefore$ number of rows added $= 5$ ✓(correctly calculates number of additional rows)

Question 6 **(5 marks)**

The curve $w(x)$ has the gradient function:

$$w'(x) = ax^2 + bx - 4$$

where a and b are constants. The curve $w(x)$ passes through the origin and has turning points at $x = -2$ and $x = 3$. Determine the values of a and b and give the equation of $w(x)$.

$$\left.\begin{aligned} w'(-2) = 0 &\Rightarrow 4a - 2b - 4 = 0 \\ w'(3) = 0 &\Rightarrow 9a + 3b - 4 = 0 \end{aligned}\right\}$$ ✓ (for both equations for a and b)

Solve on CAS $a = 2/3$, $b = -2/3$ ✓ (for a and b)

Note: only 1 mark of these 2 marks given if answers for a and b are given without equations for a and b

$\therefore\ w'(x) = \frac{2}{3}x^2 - \frac{2}{3}x - 4$

$w(x) = \frac{2}{9}x^3 - \frac{1}{3}x^2 - 4x + c$ ✓ (for expression for $w(x)$ in terms of c)

use $(0,0) \Rightarrow c = 0$ ✓ (for correctly calculating $c = 0$)

$\therefore\ w(x) = \frac{2}{9}x^3 - \frac{1}{3}x^2 - 4x$ ✓ (for final answer)

Question 7 **(5 marks)**

Peter can swim 200 $metres$ in 188.6 $seconds$. He is about to start a training program and he aims to reduce his time by 4% week on week after each week's training (that is, he wants his time at the end of the first week of training to be 4% less than his time at the start of the program and his time at the end of the second week of training to be 4% less than his time at the end of the first week, etc.).

(a) Give the following for Peter's times for the 200 $metres$ after n weeks of training:

(i) a recursive equation, (2 marks)

$T_n = 0.96\,T_{n-1}$ or $T_{n+1} = 0.96\,T_n$

$T_0 = 188.6$ $T_0 = 188.6$

(either)

✓ (for T_n or T_{n+1} recursive expression)

✓ (for T_0 value)

(ii) the equation for the n^{th} term in the sequence in terms of n. (1 mark)

$$T_n = 188.6(0.96)^n$$

✓ (for final answer)

(b) During which week of training would Peter expect his 200 $metres$ to first reach 143 $seconds$. (2 marks)

$$143 = 188.6(0.96)^n$$

✓ (for initial equation)

Solve on CAS $n = 6.78$ weeks

∴ Peter would first reach 143 seconds in 7th week

✓ (for correct final answer)

Question 8 **(8 marks)**

The displacement of a particle moving along the x-axis is given by:

$$x(t) = t^3 - 9t^2 + 24t$$

where x is in metres and t is in seconds, $t \geq 0$.

Calculate the following:

(a) The displacement of the particle after 3 seconds of motion. (1 mark)

$x(3) = 3^3 - 9(3)^2 + 24(3)$

$= 18$ m ✓ (for final answer)

(b) An expression for the velocity of the particle in terms of t. (1 mark)

$V = 3t^2 - 18t + 24$ ✓ (for final answer)

(c) The displacement of the particle at the times when its velocity is zero. (3 marks)

i.e. $\frac{dx}{dt} = V = 0$

$3t^2 - 18t + 24 = 0$

$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2$ or 4 seconds ✓ (for both times)

$x(2) = 20$m ✓ (for displacement at $t=2$)

$x(4) = 16$m ✓ (for displacement at $t=4$)

(d) The total distance travelled by the particle in the first five seconds of its motion. (3 marks)

particle turns at $t = 2$ and 4 seconds ✓ (for recognising turns at $t=2, 4$)

$0 \rightarrow 2 \rightarrow 4 \rightarrow 5$

$0\text{m} \rightarrow 20\text{m} \rightarrow 16\text{m} \rightarrow 20\text{m}$ ✓ (for appropriate working out for distance travelled)

distance $= 20 + 4 + 4$

$= 28$m ✓ (for final answer)

Question 9 **(5 marks)**

Consider the recurring decimal $0.\overline{16}$.

(a) Show that this recurring decimal can be written as an infinite geometric series with first term $\frac{16}{100}$ and common ratio $\frac{1}{100}$. (3 marks)

$$0.\overline{16} = \frac{16}{100} + \frac{16}{10000} + \frac{16}{1000000} + \ldots$$

✓ (for expressing decimal as the sum of a sequence of fractions)

This is an infinite geometric series with

first term $= \frac{16}{100}$

✓ (for showing first term $= \frac{16}{100}$)

common ratio $= \frac{16/10000}{16/100} = \frac{1}{100}$

✓ (for showing, with calculation, that common ratio $= \frac{1}{100}$)

(b) **Using your answer from part (a)**, show how $0.\overline{16}$ can be written as a fraction in its simplest form. (2 marks)

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{16/100}{1-1/100}$$

✓ (for correct substitution into formula)

$$= \frac{16}{99}$$

✓ (for final answer)

End of questions

Additional working space

Question number:______________

Additional working space

Question number:_______________